MATHEMATICS AND ITS POWER TO TRAIN THE MIND.

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The very important place that mathematics occupies in education is sufficiently evinced by the fact that it has always been judged to be an essential branch in the curriculum of every school of learning in the world. Mathematics is the science of number and quantity. It is by its very nature an abstract science, prescinding entirely from the elements of time and physical reality, and is therefore well adapted to train the intellectual faculty of the soul. Practical application to concrete examples may be an interesting presentation of the principles involved, but it is not the primary purpose of mathematics, and should not be made to obscure its original idea.

Mathematics trains a young mind to be precise, to reason, to analyze, to prove, to deduce consequences. It gives the greatest possible certainty to its pronouncements and makes the thinker an independent and personally responsible judge. It is the purpose of this article to dilate somewhat upon these statements.

Mathematics is pre-eminently the exact science. It guarantees the most absolute and the most perfect exactitude conceivable, it gives results which may be relied upon at all hazards, and upon which one may stake his fortune and his life. In return for this exactitude it demands the most scrupulous attention to details, it admits of no carelessness, no slovenliness in its operations. Strict attention to little things is its motto, and is the first lesson it teaches the student.

A want of such attention to the proper mathematical words and operations brings with it the inevitable penalty of confusion and error. Careless students are, at the very

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end of their college courses, continually struggling against elementary difficulties that they should have mastered at the very beginning. They do not understand the new matter because the professor is using words of whose meaning they have but a confused conception, and while they are thus practically driven to study all their branches of mathematics at the same time without order and without direction, there can be but one result, confusion, disgust and despair.

On the other hand an habitual attention to details and to the right use of words will soon cause the student to realize that he has made progress. All the intermediate steps in the solution of a problem become clear and connected in his mind. He sees the purpose and the reason of each one of them and of the order in which they succeed each other. Words take on specific meaning, whole operations are seen at a glance, and the method of solving problems becomes clear, rapid and interesting. A mathematical precision reacts upon the student’s literature, and gives exact expression to the idea conceived. It endows the speaker and writer with a conscious grasp of his subject, and introduces strength and vitality into his style.

The power of analysis is another gift of mathematical training. When one has habituated himself to define his terms and to scrutinize every part of his own definition, to subject his quantities to all kinds of operations, to examine his figures from every point of view, and to discuss every step in the solution of a problem, one can scarcely fail to acquire a kind of mathematical instinct which enables him to instantly bring all kinds of siege guns to the attack of a new problem, or all kinds of reagents to analyze it into its components.

Analysis means breaking up, a resolution into constituent parts. It means the apparent substitution for one quantity of a number of others, each of which may be treated in a similar manner and replaced by others. The mind is thus led into the consideration of these elements
or parts, and by studying them, into a comprehension of the original subject. Such an analysis can, in an instant, cause difficulties to vanish, it can suggest a plan of attack that will surely lead to victory, and it can demolish an adversary's argument by simply and literally taking it to pieces.

An analysis is by its very nature, an exhaustive process. When one has made a mathematical analysis, he is superior even to an assayer who has made a chemical analysis of an ore, for he knows the whole truth, and can give an entire and exhaustive account of his subject matter in such a way that absolutely nothing can be added to what he has said. This is the case even in elementary mathematics, and the student can thus learn what mathematical analysis means at the very beginning of his course. He cannot fail to wonder at the power put into his hands, for when one is in possession of the whole and entire truth, so that nothing whatever as to compel its acceptance. The mind is not a free faculty, no other view of the subject can be presented that he has not already accounted or provided for, what better school can there be in which to learn and acquire this immense, and almost infinite power of exhaustive analysis, than that of the daily practice of mathematics?

The proof of a mathematical statement is always such as to compel its acceptance. The mind is not a free faculty, when a truth is presented to it, it must necessarily accept it. Hence, when a student has truly proved a mathematical statement, he is entitled to the satisfaction of knowing that everybody in the world is obliged to accept his proof, and that when there is question of such a mathematical proof, he is on a par with the greatest geniuses of the world and can compel them to listen to him.

The certainty of a mathematical proof is of the highest order, it is metaphysical or absolute. It is so great that one may safely stake his fortune and his life upon it. No fear
need be entertained that the proof will ever be undermined. I am speaking here of the simple problems that a student meets in his course, not of the long and complex researches of professional mathematicians, which, as history tells us, have often, even after a lifetime of the greatest labor, been shown to be false and untenable. But even in these cases the fault was with the human element, which is necessarily limited and not omniscient, and not with the science of mathematics, which must be said rather to have scored another victory in thus disclosing to us the mistakes of individuals.

The certainty of mathematical conclusions effectually eliminates the principle of authority. When one has proved a proposition, he need not be concerned about the judgment of others. It is truth to him, and must, therefore, be truth to the world, no one can contradict it. Hence, the authority of others in confirming our statement is needless, it is implicitly contained in our proof, we have it already, we have forced it by our own power. While on the one hand reliance upon the answer to a problem may be pardoned in a beginner on account of his inexperience, still on the other there is a real need of this pardon, because the student is making a mistake in principle, he is replacing metaphysical or absolute certitude, which is the highest of all, and which it is the nature of mathematics always to give, by moral certitude, the lowest of all, by taking the word of the author and of the printer.

This is the hardest lesson a student has to learn, and many do not learn it at all, this rejection of all authority, this true liberty of judgment, and this independence of the answer book. But it is the grandest lesson that mathematics has to impart. It makes a man out of the student, it teaches him to sit in judgment on the world, and shows him how to lead the thought of his fellows. It prepares him for one day passing a correct judgment upon an accused, or upon the work of an employee. It tends to make him a just
arbiter of human differences. It gives him that true inde­
pendence that befits a human being. And, withal, it teaches
him honesty and uprightness of character, and then a fear­
less facing of all consequences. When one is absolutely con­
vinced of the truth of his statement, when one is in posses­
sion of the entire truth, and when one knows that the whole
world cannot differ from him, why then should there be any
fear, why hesitate, why not pronounce judgment and let
the consequences be what they list?

On the other hand the practical errors that flesh is heir
to teach the student prudence and humility. Long problems
develop his endurance. Continued practice sharpens his
wits and shows him many abbreviated processes. Then
many elegancies appear in his work and in his methods.
And, if his practice is sufficiently extensive and his applica­
tion sufficiently prolonged he will begin to see some of the
beauties of the science, he will get some glimpses of its
infinite possibilities, another world, of which before he had
not even an idea, will open up before him, and he will then
realize that there are charms and fascinations even in
abstract mathematics which are absolutely sealed treasures
to all but its most faithful and persevering students.

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