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AN AGENT-BASED MODEL OF THE INTERNATIONAL NUCLEAR SYSTEM

By

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A Thesis

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ABSTRACT

Despite efforts from the Strategic Arms Reduction Treaty and the Nuclear Non-Proliferation Treaty, the last decade has seen a growing number of states in possession of nuclear weapons. As states enter and exit the nuclear proliferation arena, a set of rules (implicit and otherwise) guide each action. These actions have effects on the reactions of other actors. I investigate this phenomenon using an Agent Based Model to represent the interactions among seven states and the four calculation-based attributes: cooperation, acquisition, proliferation, and use. Each state is affected differently by the actions of others and defined by a slightly different set of boundaries. By calculating each attribute simultaneously, states are able to compare outcomes and act on the optimal one. The model simulates the actions and reactions of these states within the context of the set boundaries.
ACKNOWLEDGEMENTS

Had it not been for the assistance and encouragement from a number of individuals, this thesis would not have been possible.

First, I extend my deepest gratitude to Dr. Terry D. Clark, his ready advice and enthusiastic support is present on every single page of this thesis. Through mentoring, he has helped me to grow as a scholar and as a person.

A special thanks goes to Dr. Mark J. Wierman, who spent a great deal of time formatting the computer code for the model presented in this paper.

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NUCLEAR INSTABILITY

Despite the difficult process in developing and maintaining nuclear weapons, possession of said weapons continues to be a topic of importance to many states. As a result, much work has been done in the field of international nuclear system modeling and simulations in an effort to predict the movements of states regarding nuclear weapon development. However, models have failed to represent the dynamic and ever-changing global structure. Striving to understand the complex system, I first study the frequently asked question on what motivates a state to pursue nuclear weapons. I then ask how a state’s decision regarding their nuclear activities affects the calculations made by others. I use an agent-based model to simulate, in real-world time, the choices a state makes along with the reactions of surrounding states.

Given the widespread, and dangerous, history surrounding nuclear weapons, the smallest possibility they are present in a state can begin rounds of heated worldwide debates. The weapons play a major role in how a state interacts with another on an international scale. Scholars have attempted to explain a state’s decision to obtain nuclear technology in hopes of predicting, and ultimately, preventing the spread of all nuclear materials. Most theories fall into one of three categories; states are able to pursue nuclear weapons, states are willing to pursue nuclear weapons, or states are able and willing to pursue nuclear weapons.
For those scholars that suggest nuclear abilities as the key indicator of a state’s proclivity in seeking nuclear technology, they often cite the difficulty states face when attempting to support a nuclear program. As there is no evidence of states obtaining nuclear weapons through transfers with other states, scholars argue that a program is a prerequisite to developing a weapon. Three theories support the ability paradigm: production capability, economic capacity, and knowledge.

*Production Capability:* There is a lengthy list of materials and resources a state would need to develop a nuclear weapon program. It can include everything from the fuel cells, to the facilities where the program is maintained, to the scientists (engineers, physicists, chemists, etc.) working with the materials (Jo and Gartzke, 2007; National Security Office, 2007). In addition, most do not have the equipment needed to maintain the plutonium and highly enriched uranium used in nuclear weapon development. Because of the protections and international laws regulating the sale, transfer, and production of the equipment, it can be more difficult for some states to even obtain the essential material.

*Economic Capacity:* Nuclear programs are not developed without significant financial means. It is an expensive endeavor, one taken by few states, as many cannot foot the costs required to nuclearize (Gartzke and Jo, 2007; Elhefnawy, 2008; Kroenig, 2009;
Weiner, 2009). The start-up and maintenance costs are exceptionally high, so high that the only program costing the United States more in the long run is Social Security. Libya spent a total of $100 million and thirty years attempting to build a nuclear bomb with little success (Mueller, 2010). For these scholars, having the funds is necessary to any type of nuclear technology development. However, there are those that argue that as technology improves, the financial burden is no longer as great as it once was (Bahgat, 2006b; Jackson, 2009). As a result, some argue that economic capacity is less likely to be an indicator for nuclearization in the future.

Knowledge: Knowledge is the foundational requirement for any nuclear program. A state can have all the equipment and all the funds, however, without knowledge of the processes and the correct application, the equipment is worthless. The level of knowledge and experience necessary for an effective nuclear program is not always readily available. In Libya’s case, money was not the only obstacle to nuclear weapon development, the lack of extensive nuclear knowledge resulted in the eventual failure. Given this fact, some argue that a state who maintains close relations with a nuclear state is more likely to become nuclear because of the opportunity for knowledge diffusion; that is, the spread or sharing of knowledge and information from one entity (in this case the state) to another (Gartzke and Jo, 2007). They often point to the case of Syria and North Korea. Evidence presented
by the Central Intelligence Agency (CIA) to the U.S. Congress in 2008 confirmed that the two states had shared scientists as well as the plans and operations for the nuclear reactor architecture. In the end, Syria created a nuclear program very similar to the one present in North Korea (CIA Report to Congress, 2008).

While scholars agree that ability is important, they argue that a state would not have a nuclear program if they were not first willing to do so. With that line of thought, many theories have been grouped into the category of studying why a state would be willing to make the sacrifices, and take on the risks, necessary to develop a nuclear weapon. Seven main theories are evident in this literature: conflict, nuclear threat, defense agreements, diplomatic isolation, domestic influences, democracy, and power status.

Conflict: The Soviet Union began nuclear research in the 1930s, however, not until 1949 were they able to test an atomic bomb. Concerned with the Communist country’s substantial nuclear arsenal, the U.S. government debated whether the size and strength of its own program was sufficient. The advent of the Cold War saw an increase in the number of nuclear weapons created and tested along with a persistent tension between the two states. The situation became very serious in 1962 with the Cuban Missile Crisis; it was the closest any two countries had gotten to a nuclear exchange. Even more seriously, it was twenty years after the first bomb was dropped in Japan and
science had vastly improved, nuclear bombs had the potential to be twelve times greater than that of the 1940’s version (Mueller, 2010). Altogether, the Soviet Union and the U.S. had 70,500 nuclear weapons; enough to destroy the entire world 25 times over (ICAN).

Studying the events of the Cold War, scholars argued that the increase in nuclear weapons was a result of the security concerns each state had at the time (Sagan, 1994; Arbatov, 2006; Singer, 2007; Lund, 2009). States with nuclear weapons often deter enemies from aggression for fear of nuclear retaliation. This situation can be further exacerbated if the nuclear state feels there is a greater lack of security due to military weakness (Campbell, 2002; Mearsheimer, 1984). However, conflicts for nuclear states are more likely to be shorter as well as occur with less frequency, giving a greater feeling of security overall (Gartzke and Jo, 2007). Knowing that, states are more likely to develop nuclear weapons in hopes of increasing conflict success along with reducing conflict involvement.

*Nuclear Threat:* Beyond normal conflict, states that have nuclear rivals are argued to be more likely to acquire a nuclear program. This is firmly evidenced by the arms race between the U.S. and the USSR (Foran and Spector, 1997; Marwah, 1981; Singh, 1998). A state would not be content to avoid conflict with a nuclear rival for fear of attack. By having their own program, a rival state can equally deter the enemy, creating a stalemate or a systemic equilibrium.
However, others claim nuclear rivals become more of a threat when an opponent begins to seek nuclear weapons in defense (Bueno de Mesquita and Riker, 1982; Cropsey, 1994; Mandelbaum, 1995). Scholars reason that as nuclear rivals observe opponents developing nuclear weapons, they will act in an effort to prevent others from gaining any increased weaponry. Contrary to the previously discussed theory, these scholars argue that states with nuclear rivals are less likely to acquire nuclear weapons.

In addition to the state-to-state rivalry, scholars contend that states are more likely to seek nuclear weapons when there is an increase in the number of nuclear states in the system. In 1952, the United Kingdom became the third state to test nuclear weaponry. France followed a few years later with the testing of an atomic weapon in early-1960. China detonated its first atomic bomb in 1964. And, in 1998, India and Pakistan began developing and testing their own nuclear power. North Korea conducted tests in both 2006 and 2009 despite the backlash from the international community. Israel has never conducted tests, however, they have long been suspected to possess nuclear weapons (ICAN).

*Defense Agreements*: Scholars argue that state participants of defense agreements are much less likely to seek nuclear weapons. Nuclear-weapon-free zones (NWFZ) “commit themselves not to manufacture, acquire, test, or possess nuclear weapons (Arms Control
Association).” These states bond together, often building coalitions with a nuclear state in order to protect themselves in the event of a nuclear strike. Five regions, or 89 states in total, have signed NWFZ agreements: Latin America (1967), the South Pacific (1985), Southeast Asia (1995), Africa (1996), and Central Asia (2006).

Furthermore, the Nuclear Non-Proliferation Treaty (NPT) prohibits a nuclear attack on non-nuclear states. This stipulation allows non-nuclear states to experience a sense of security without the ownership of nuclear weapons. However, there are varying theories regarding the NPT agreement. On one hand, studies have shown that member states are much less likely to seek nuclear weapons (Simpson and Howlett, 1995). This could be due, in part, to the prohibition on nuclear development by non-nuclear states. Additionally, the recognition of only five nuclear states, along with the wording of the NPT, which asks nuclear states to “pursue negotiations in good faith on effective measures relating to cessation of nuclear arms race at an early date and to nuclear disarmament, and on a treaty on general and complete disarmament under strict and effective international control,” further deter non-nuclear states (Nuclear Non-Proliferation Treaty, 1970). The agreement also prohibits the transfer of nuclear weapons, nuclear explosive devices, or their technology to non-nuclear states. Furthermore, agreements like the NPT have the ability to change the behavior of members as it gives states a place to form social
connections with likeminded states (Finnemore and Sikkink, 1998; Wendt, 1995). On the other hand, there are those that have claimed the NPT is not an effective deterrent to nuclear weapon acquisition due to lack of rule enforcement as well as a lack of control over non-member states (Braun and Chyba 2004).

*Diplomatic Isolation:* Some theories have focused on the states that lack the average number of social connections one may see in most other states. They argue that pariah states, or “rogue” states, are more likely to build nuclear programs due to their own security concerns. They lack the safety in numbers other states are able to develop as well as any security umbrella agreement. In addition, pariah states are less deterred by the economic sanctions placed on nuclear developing states because of their position in the periphery or outsider status. These types of punishments do not carry as much weight as they do for states in the core (Lake, 1994; Harkavy, 1977, 1981).

*Domestic Influences:* For some states, nuclear weapons are a source of national pride. States have seen a rise in nationalism as a result of nuclear program development. For this reason, states that experience increasing domestic sentiments towards nuclear weapons or a greater military power will be more likely to seek to acquire nuclear weapons. Additionally, states that experience internal unrest and decreasing popularity in the government are also more likely to
develop a program in efforts to control society through a show of dominance (Gartzke and Jo, 2007).

Democracy: Some scholars have focused on the large number of democratic nuclear states. They claim that democracies or partial democracies are more likely to seek a nuclear weapon program (Gartzke and Jo, 2007). Much of this literature is based on the powerful democratic states that escape international backlash when acquiring nuclear weapons. Some scholars disagree with this analysis and, instead, focus on the peaceful nature of democracies along with their interdependence among each other, decreasing the number of states with security concerns or nuclear arms ambitions (Chafetz, 1993; Singh and Way, 2004). Additionally, the spread of democracy leads states to want in to the global economic-political core, further reducing the number of states that are willing to pursue nuclear weapons for fear of being left out.

Power Status: Scholars have debated the propensity for major international and regional powers to nuclearize. They claim that states with a high power status are more likely to seek nuclear programs and weapons (Gartzke and Jo, 2007). This is in large part due to their ability to withstand the international relations ramifications as well as the NPT recognition of the five largest powers’ right to nuclearize. Powerful states can accept a certain amount of risk because they can recoup any losses when the benefits are realized.
This literature is comprehensive on explaining why a state may choose (or not) to pursue nuclear weapons, however, it does not completely answer the question of state interactions and how they might affect the entire system (and vice versa) rather than the relations between only two or three states. For the purposes of simulating this environment, two foundational assumptions are used to determine the make-up of the model. Firstly, states are more sensitive to changes in nuclear states rather than in the more cooperative states. States feel more secure with cooperation because the likelihood to acquire, proliferate, and use nuclear weapons is decreased. States that decide not to cooperate are more of a risk to the international system as a whole. There is a fine line between acquisition, proliferation, and use. As a result of the risks states feel, their cost/benefit/risk calculations reflect the move to a more active defense/offense posturing.

Secondly, states are more sensitive to changes made in states that belong to the same network. States are clustered based on interactions. These interactions include economic and diplomatic exchanges. In many instances there are mirroring interests between the two states. As a result, when states calculate their cost/benefit/risk, decisions and actions by the states around them have a larger impact than those at a greater social distance.

In order to answer the question, what conditions cause a state to pursue nuclear weapons and to what extent do those decision
calculations affect the nuclear system, I use an Agent-based Model (ABM). The ABM is a flexible tool and it effectively depicts the global system. The two foundational assumptions mentioned above provide the rules used to direct state-to-state interactions. As a state makes decision calculations, the states around it will adjust their own calculations in order to achieve the best optimal outcome. This process is never ending as calculations are modified continually.

Testing the model, I begin with seven states. Each state will calculate decisions based on four attributes: cooperation, acquisition, proliferation, and use. Two sets of tests are conducted, one for a nuclear state and one for a non-nuclear state. Variables are kept to a minimum in order to best observe the outcomes. The results show a greater sensitivity to the change of status quo in states that are already nuclear. These calculations seem to have a greater affect on the entire system rather than just the states within the nuclear state’s social circle. However, the overall affect of a decision is the same, the system just reacts at a faster pace for changes made in the nuclear state. Further tests with additional variables can produce a very different system than the original. The first adjustments made to the state can play a large role in the future of nuclear weapons.

The state’s status in a nuclear world and any changes made to that status are determined by a state’s environment as well as its interactions in the global system. Barring any fundamental changes to
the status quo, states that are currently cooperating are more likely to continue doing so; given the protection they receive at very little cost to themselves. Slight changes to the status quo could affect states at the acquisition stage. They can make choices to cooperate if the cost becomes too high or if the motivation is no longer present. They could also choose to proliferate if the cost is not a deterrent and the motivation is still present. Those that have proliferated are more likely to remain with the status quo as the cost is less of an issue after start-up, in some cases it could cost those states more to disarm, and there is even less external motivation for disarmament. The constant change in the nuclear system and the interactions between states will determine the future of an individual state, further affecting the rest of the system. This occurs in a never-ending, cyclical manner.

The topic of a nuclear future remains a discussion point for the United Nation’s (UN) General Assembly. In just the last few years it has passed a resolution in support of disarmament and regulating policy. As the UN monitors the signatories of the Non-Proliferation Treaty, it performs the security checks required by members. Keeping states from taking action that starts a chain reaction of events is an important part in non-proliferation. States seeking deterrence policies, much like the United States, will have a better understanding and a method for tailoring their efforts if they know the catalysts for nuclear proliferation (Geller, 1990; Wareham, 2005).
Each state must make one of four choices related to the nuclear system: *cooperation, acquisition, proliferation, or use*. I define *cooperation* as the willingness of states to take part in voluntary monitoring and inspections of all nuclear facilities as well as continued support in the effort to reduce nuclear weapons. The term *acquisition* refers to states that obtain or gain possession of any materials or technology in relation to a nuclear program. *Nuclear proliferation* is defined as the construction and presence of a fully developed nuclear weapons program up to the possession of a nuclear weapon within a state. A nuclear weapons program can include the presence of nuclear materials, delivery devices, or the scientists and facilities used to maintain, develop, or design any weapons. A state in possession of a nuclear weapon will be fully in charge, with the ability to arm and use it (Gartzke and Jo, 2007). Finally, I define *use* as the arming, launching, or releasing of nuclear weapons, to include any testing\(^1\).

While the role of each individual state is important, this study will group the states into categories meant to represent the different orders found in the international system. This is used to determine one type’s effect on another.

In the sections that follow, I develop an Agent Based Model (ABM) in order to observe the affects of *cooperation, acquisition, proliferation, and use* in regards to nuclear weapons. Furthermore, it

\(^1\) The four choices as well as their definitions were developed from the 2009 GISC Nuclear Stability Framework Assessment Workshop
demonstrates the possible future of the international system given an action by one state and the change in the system as a whole.

THE AGENT-BASED MODEL

Early agent-based models (ABM) were not well understood because of the widely held belief that the sum of the parts must equal the whole, regardless of the fact that scientists were aware of systems where this was not the case (Heath, 2010). These nonlinear systems proved to be too much for the tools available at that time.

Despite the issues surrounding early ABMs, the ABM as we know it today was developed using the Von Neumann architecture. The Turing Machine was an early “computer” that ran on several hundred cards with rules that would guide the processes of the computer. This machine would aid in the development of rule setting for artificial systems. The Von Neumann design was a basic matrix that allowed interactions to occur between neighboring squares (agents) using a series of very specific rules to guide the interactions. The idea was then molded to include a model with very simple rules. The rules included a diffusion effect; one agent would light up, that light would affect those squares that have common
borders and then the squares neighboring those. In Figure 1\textsuperscript{2}, the yellow square was the first one lit, its neighbors (the green squares) lit up as a secondary and resulting action. However, the impact of the effect would diminish the further it moved away from the originating square. For example, the yellow light in the center would experience a stronger energy than the green squares as they receive the byproduct from the yellow square, rather than experiencing the original, full effect. This concept is used in similar fashion for the Moore architecture; however, the rules for diffusion include the squares angled to the original. Like Figure 1, Figure 2\textsuperscript{3} demonstrates the primary and secondary reaction of a lit square. The difference is in the rule that determines the definition of a neighbor. The Moore architecture includes neighbors as the angles as well as neighbors sharing borders.

The ABM concept was not used solely in computer simulations; an experiment of the Prisoner’s Dilemma brought the concept to life. Robert Axelrod held a tournament where participants were required to follow a set of rules for each choice they made (Axelrod, 1997). Given these rules, Axelrod was able to simulate and examine the cultural phenomenon that forms when certain rules are

\textsuperscript{2} http://jasss.soc.surrey.ac.uk/13/2/10.html  
\textsuperscript{3} http://jasss.soc.surrey.ac.uk/13/2/10.html
set. The ABM soon moved to the “hard” sciences, including the field of biology. Many scholars used the ABM to simulate the biological life, or artificial life. The models were used to observe phenomenon where experimentation could not be replicated.

While ABMs have proven useful in scholarly studies, they have also been used to predict flows (evacuation, traffic, etc.), changes in economic markets, or organizational developments. One such example is the fire escape model. A number of people (agents) are put on one side of the wall (object). A fire alarm is set off and the agents must escape using one door put in the wall. The speed is set based on human ability and the size of the door will determine how many can pass through at once. In this example, the environment is the space broken up by one wall. The environment can be changed depending on the simulation requirements and the modeled building’s architecture. This model was run again with a column placed in front of the door. The model was able to determine that the column would naturally filter people through slowly so that they were not pushing and getting stuck at the door. More people were able to escape (Bonabeau, 2002).

I use an ABM in order to demonstrate the affects any system change may have on the nuclear system status quo. As a complex issue, the spread of nuclear weapons in the international system can be understood and represented by an ABM. The basic agent-based
model simulates a set of “agents,” all of which interact in an “environment”. Agents are autonomous, decision-making actors that are able to move or remain stationary in an environment. A set of rules governs the agents’ actions as well as reactions to other agents and objects. The environment is the confines within which agents are placed. Environments are adjustable depending on the settings and requirements of the ABM. In an ABM, the sum is greater than its parts, the ultimate goal of the model is not to see the movements of each agent but rather to see the system outcome as a whole (Bonabeau, 2002).

ABMs can have very simple designs while modeling a very complex situation or they can have a very complex design, modeling even more complex situations. Either way, the benefits of agent-based modeling are apparent: ABMs capture phenomenon in real-time, they allow for the simulation of a natural situations, and they are flexible. As an ABM models the behavior of the agents, the entire model shifts. In the end a picture emerges from “the bottom up (Bonabeau, 2002).” Because the ABM is able to change with the agents it produces results that are much more natural and better comparisons for plausible situations. The flexibility in the model is due to the ability of agents to shift behaviors given certain situations. In addition, they are able to adapt and learn throughout the simulation. These three reasons make
ABMs popular and useful for dynamic studies, including a study on the nuclear system.

NUCLEAR SYSTEM SITUATION

The following is an overview of the agent-based model I use to represent that system. The source code, written in Java language, is found at Appendix 1. I use seven agents to represent the international system, each agent has four attributes: cooperate, acquire, proliferate, use. As an agent calculates the costs and benefits of each attribute, the attribute with the largest payout is acted upon. The attribute identity calculation is a number that will fall between zero and one. A zero indicates very little benefits received from that action, a one indicates a maximum benefit received from that action. The calculations of each state’s attribute are independent of the values of the other three attributes. Therefore, factors taken in to account in one attribute may be calculated differently for another. The equation used as a foundation to this model is found at Appendix 2.

As a result of the final calculations, each agent is displayed as one of four colors, green, yellow, orange, or red, indicating the highest calculation outcome as cooperating, acquiring, proliferating, or using, respectively. Initial colors are set based on the given international system at setup. Colors adjust automatically as the simulation is run.
Agents are placed at a set distance from each other (this is not done visually but written into the underlying code); the distance is not geographical, but rather a social relations distance. Visually, this can be seen in Figure 3. All states have one edge connecting it to every other state. A number (ranging from 0 to 1) on each edge signifies the distance between those two states. A 0 indicates a close social distance while a 1 indicates the maximum social distance, this can include states that have very little to no social interaction at all. States that interact more often will have a closer social relations distance than

![Figure 3: Agents with distance lines](image-url)
those that have little interaction. Actions by the state are more likely to affect those that are closer socially than those that just border geographically. For example, states that are closely linked by economic ties will feel the effect at a much greater rate if the other begins to experience economic decline than those that have very little economic connections (similar to an economic crisis in Greece and the effect it can have on the European Union as a whole). Much like the Von Neumann and the Moore architectures, the agents surrounding the focus agent will be affected by any change at a greater strength than those further out in the periphery.

Each simulation begins with the initial setup. States are placed in a circle with the lines labeled with the social distance. The three charts to the right of the display screen indicate the distance and attribute levels; the display screen can be seen in Figure 4. The very top of the screen displays the action keys, such as a “Go” button or a “Reset” button. The first

![Figure 4: Nuclear ABM Display of State Attributes, Connection Distances, and Attribute Diffusion Rates](image-url)
matrix indicates the current level of each attribute for each state. The second gives the previous levels. Figure 4 is the initial setup so the matrices are the same. The third matrix shows the pre-designated social relations distances, while the fourth gives the diffusion rate for each attribute. The diffusion rate determines the sensitivity of one action on the calculation on the others. Figure 5 shows the entire screen at initial setup.

![Figure 5: Full Nuclear ABM Display](image)

At the start of each test an adjustment is made manually in the first matrix to one attribute of a single agent. That change is recognized by the system and the simulation begins. As the “Go” button is pushed, the program distinguishes the changed state and it initiates the predetermined rules that determine how the rest of the system reacts. Depending on the attribute that was adjusted, the system will then use the varying rate of diffusion for all other states.
and their attributes. The smaller the diffusion rate between attributes, the smaller the effect. The smaller the distance between states, the greater percentage of diffusion.

TESTING

In order to get an accurate picture of the effects of small and large changes in several states, I test the model sixteen times at a fixed level of change (with a positive to negative shift) in two states and in all attributes. Each test begins with the exact same setup. The states’ attributes are set at the same initial values as well as all social distances. Both of these values can be changed at any time, however, if the model is “Reset”, the values will revert back to the originals.

All simulations begin by a manual change to one state’s attribute value, for this explanation State X. The difference between the original and the new value is recorded. The change to the system starts when the “Go” button is pushed. Every time the “Go” button is pushed, the simulation will run through the selected iterations. As the first iteration cycles a percentage of the difference made to State X is diffused to the other states in the system (State Y), the level at which this is done is the diffusion rate. A positive or negative change is expected depending on the attribute that was changed in State X as well as the observed attribute in State Y. The percentage of that value depends on the social distance between State X and State Y, the closer the two states, the greater the change in State Y.
Following the completion of the first iteration, the new values are recorded in the table. The second iteration will record the changes to each state in the system and cause fluctuations in the other states accordingly. Once again, the results are recorded and the simulation continues. As the values change, the color of the states will change visually to reflect the highest attribute value (green for cooperation, yellow for acquire, orange for proliferate, and red for use). As the simulation is completed, results from the entire test are represented in a spreadsheet as well as a graph.

Each test keeps the variables to a minimal so that the effect of each change can be documented with fewer independent variables. This will show a better representation of the sensitivities from and to each attribute. The description of each test is listed below.

<table>
<thead>
<tr>
<th>TESTS FOR STATE WM</th>
<th>TESTS FOR STATE RA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: +0.02 Cooperate</td>
<td>9: +0.02 Cooperate</td>
</tr>
<tr>
<td>2: +0.02 Acquire</td>
<td>10: +0.02 Acquire</td>
</tr>
<tr>
<td>3: +0.02 Proliferate</td>
<td>11: +0.02 Proliferate</td>
</tr>
<tr>
<td>4: +0.02 Use</td>
<td>12: +0.02 Use</td>
</tr>
<tr>
<td>5: -0.02 Cooperate</td>
<td>13: -0.02 Cooperate</td>
</tr>
<tr>
<td>6: -0.02 Acquire</td>
<td>14: -0.02 Acquire</td>
</tr>
<tr>
<td>7: -0.02 Proliferate</td>
<td>15: -0.02 Proliferate</td>
</tr>
<tr>
<td>8: -0.02 Use</td>
<td>16: -0.02 Use</td>
</tr>
</tbody>
</table>
A positive or negative 0.02 change is adjusted for the appropriate attribute, in the appropriate state. Each test is run individually and the results are recorded and graphed in Appendix 3 and 4. The only variable for the tests is the positive or negative change set at the beginning. Beyond the initial change, both tests are run for 500 iterations.

RESULTS

The results from each test are depicted below in Table 1. Each result is given with four numbers \( (w,x,y,z) \), the first \( (w) \) gives the number of total states cooperating, the second \( (x) \) acquiring, the third \( (y) \) proliferating, and the fourth \( (z) \) using. All tests correspond to the list of tests mentioned above. Tests results marked with an asterisk (*) demonstrate points where the simulation converged or where the system could no longer make changes (often due to one state reaching the maximum or minimum resulting in no more changes being made to the system).

<table>
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<tr>
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</table>

State WM leans towards the cooperative side of the spectrum while State RA tends towards the nuclear end; however, both states belong to the same cluster. Due to this, changes to each state should cause similar, if not the same, results in the entire system. As Table 1 shows, that was not always the case. State RA, as the more nuclear state, tended to cause greater sensitivity to the changes when the attribute was adjusted. However, while the system seemed to be more sensitive to a change in State RA versus any changes in State WM, nearly all of the adjustments resulted in the same reactions by the end of 500 iterations. Results from tests 2 and 10 are graphed below in Figures 6 and 7 for comparison.

![Results of Test 2](image)

*Figure 6: Results from Test 2, +.02 Acquire*
While the graphs appear to be very similar, the change in Test 10 shows a greater sensitivity when a more nuclear state increases its acquisition 0.02. Both tests end with all seven states acquiring, however, the time it takes to do so is greater when the change occurs in a more cooperative state. When comparing Tests 8 and 16, a negative adjustment to a states calculation to use nuclear weapons, we see no change at all in the cooperative State WM’s system. This could be due to the little effect it had overall in State WM. While the same change in the more nuclear leaning State RA caused an increase in cooperation among the system.

CONCLUSION

During the entire simulation, not one state approached proliferate or use. The model was set up so that states were not predisposed to those variables and as a result of a single change, the states were not able to see
affects that would cause them to calculate high outcomes for proliferate or use.

In these tests, only one value, in one attribute was changed and at only a single moment in time. Given the parameters of the ABM, values can be changed several times which would be a much more accurate portrayal of the international system. These changes do not need to be made at the very beginning of every test; rather, values can be changed as the test is running. This would allow for additional variables (internal and external to a state) to affect the system.

Additionally, values can be adjusted in the distance or diffusion category. It is possible to change the distance between two states as the model is running. The social relations between states are continuously changing and that must be represented in the model. The strength of the diffusion rate for each attribute also changes through time. The affect of a change to proliferate, for example, may result is a more sensitive change to other states. Much of this depends on the system and the events of the time. While most other models cannot effectively capture more than one system throughout time, the ABM is able to capture as many systems as a user is willing to add.

The changes mentioned above are necessary in further tests in order to see how the model stands up in an even more accurate, and more discrete, trial. While single variable changes were effective for initial testing, it cannot really give a picture that is similar to what one might see
in the international system. Several changes in one simulation may show a balancing act between several states in an effort to avoid any large and damaging conflict. The same can be said for the social connections. As states become more dangerous to the system, others may distance themselves in order to avoid any consequences by association. Additional tests can also support, or not, the foundational assumptions.

Given the importance of nuclear weapons in the international system and the actions that states take with them, the question of why a state would seek nuclear weapons and how that decision affects the rest of the system is important. The ability to predict these events, especially catalysts, is critical to maintaining security. When those catalysts can be predicted, they can also be avoided. As the U.S. makes policy for deterrence strategies, knowing what chain reaction a decision could cause can lead to success of help avoid disaster.

The ABM is an effective tool to doing just that. An ABM can adapt to match the real world resulting in a more accurate picture for prediction. The most difficult part is creating the assumptions that form the foundation of the model. However, once that has been completed, the model can run with factors added throughout time. Variables can shift to represent that system at a specific time, creating a model that can shape policy,
APPENDIX 1: MODEL SOURCE CODE

package seven;

public class ModelSeven {

    Diffusion diffusion;
    Union originalUnion;
    Union currentUnion;
    Union previousUnion;
    Union deltaUnion;
    boolean deltaZero=true;

    public Diffusion getDiffusion() {
        return diffusion;
    }

    public void setDiffusion(Diffusion diffusion) {
        this.diffusion = diffusion;
    }

    public Union getPreviousUnion() {
        return previousUnion;
    }

    public void setPreviousUnion(Union previousUnion) {
        this.previousUnion = previousUnion;
    }

    Connections con;

    public ModelSeven(Union un, Connections con, Diffusion dif) {
        super();
        this.originalUnion = un;
        this.currentUnion = new Union(originalUnion);
        this.previousUnion = new Union(originalUnion);
        this.deltaUnion = new Union(originalUnion);
        this.diffusion = dif;
        this.con = con;
        deltaZero=true;
    }

    public ModelSeven() {
        super();
        this.originalUnion = new Union();
    }
}
this.currentUnion = new Union(originalUnion);
this.previousUnion = new Union(originalUnion);
this.deltaUnion = new Union(originalUnion);

this.con = new Connections(originalUnion.getSize());
this.diffusion = new Diffusion(State.getNumAttributes());
deltaZero=true;
}

public Union getOriginalUnion() {
    return originalUnion;
}

public void setOriginalUnion(Union originalUnion) {
    this.originalUnion = originalUnion;
}

public Union getCurrentUnion() {
    return currentUnion;
}

public void setCurrentUnion(Union currentUnion) {
    this.currentUnion = currentUnion;
}

public Connections getConnections() {
    return con;
}

public void setConnections(Connections con) {
    this.con = con;
}

public Union getDeltaUnion() {
    return deltaUnion;
}

public void next() {
    if(deltaZero) return;
    deltaUnion = new Union(currentUnion);
    //Union temp = new Union(currentUnion);
    double d = -1.9,deltaSum=0;
    int n = previousUnion.getSize(), m = State.getNumAttributes();
    int i, j, k, l;
for (i = 0; i < n; i++) // state from 
{
    for (k = 0; k < m; k++) // Attribute changed
    {
        d = currentUnion.getState(i).getAttributeValue(k) -
            previousUnion.getState(i).getAttributeValue(k);
        deltaUnion getState(i).setAttributeValue(k, d);
        deltaSum+=Math.abs(d);
    }
}
if(deltaSum<0.001)
    deltaZero=true;
//previousUnion = currentUnion;
previousUnion.copy(currentUnion);

double delta_jl, con_ij=0, attrib_ik, diffusion_lk,
change_ik,diffusion_k=0,con_i=0;
for (i = 0; i < n; i++) // State changed
{
    for (k = 0; k < m; k++) // Attribute changed
    {
        change_ik = 0;
        con_i=0;
        attrib_ik = previousUnion.getState(i).getAttributeValue(k);
        for (j = 0; j < n; j++) // State affecting i
        {
            diffusion_k=0;
            for (l = 0; l < m; l++) // Attribute affectink k
            {
                con_ij = (1 - con.getCon(i, j).getWeight());
                delta_jl = deltaUnion.getState(j).getAttributeValue(l);
                diffusion_lk = diffusion.getDif(l, k).getWeight();
                diffusion_k+=Math.abs(diffusion_lk);
                change_ik += con_ij  *delta_jl* diffusion_lk;
            }
            con_i+=con_ij;
        }
        d=attrib_ik + change_ik/diffusion_k/con_i;
        d=Math.max(0,d);
        d=Math.min(1.0,d);
        currentUnion.getState(i).setAttributeValue(k, d);
    }
}
void reset() {
    this.currentUnion.copy(originalUnion);
    this.previousUnion.copy(originalUnion);
    deltaZero=false;
}

public boolean isDeltaZero() {
    return deltaZero;
}

public void setDeltaZero(boolean deltaZero) {
    this.deltaZero = deltaZero;
}

}

package seven;

import java.util.ArrayList;
import java.util.Arrays;
import java.util.Vector;
import com.csvreader.CsvReader;

public class Union {
    ArrayList<State> states = new ArrayList<State>();

    public int getSize() {
        return states.size();
    }
    public void setState(State s,int i){
        states.set(i,s);
    }
    public State getState(int i){
        return states.get(i);
    }
    public Union(){
        try {
            readCSV(""");
        } catch (Exception e) {
            // TODO Auto-generated catch block
            e.printStackTrace();
        }
    }
}

import java.util.List;
import java.util.ArrayList;
import java.util.Arrays;
import java.util.Vector;
import com.csvreader.CsvReader;

public class Union {
    ArrayList<State> states = new ArrayList<State>();

    public int getSize() {
        return states.size();
    }
    public void setState(State s,int i){
        states.set(i,s);
    }
    public State getState(int i){
        return states.get(i);
    }
    public Union(){
        try {
            readCSV(""");
        } catch (Exception e) {
            // TODO Auto-generated catch block
            e.printStackTrace();
        }
    }
}
public Union(String s) {
    try {
        readCSV(s);
    } catch (Exception e) {
        // TODO Auto-generated catch block
        e.printStackTrace();
    }
}

public Union(Union un) {
    for (int i = 0; i < un.getSize(); i++)
        states.add(new State(un.getState(i)));
        //this.vars=Arrays.copyOf(un.vars, un.vars.length);
}

public void copy(Union un) {
    for (int i = 0; i < un.getSize(); i++)
        states.get(i).copy(un.getState(i));
}

public void readCSV(String sDataFileName) throws Exception {
    Vector<String> colheaders = new Vector<String>();
    CsvReader reader = null;

    if (sDataFileName.equals"")
        reader = new CsvReader("CAPU.csv");
    else
        reader = new CsvReader(sDataFileName);
    reader.readHeaders();
    String Test = reader.getRawRecord();
    if (!Test.contains",")
        reader.setDelimiter('t');
    colheaders.addAll(Arrays.asList(reader.getHeaders()));

    int n = colheaders.size();
    int k = 0;
    double d;
    String s;
    String sd;
    State.setNumAttributes(n - 1);
    for (int i = 0; i < n - 1; i++)
        State.setAttributeName(i, colheaders.get(i + 1));
while (reader.readRecord())
{
    s = colheaders.get(0);
    sd = reader.get(s);
    State st = new State(new Integer(k), sd);
    for (int i = 0; i < n - 1; i++) {
        sd = reader.get(State.getAttributeName(i));
        d = Double.parseDouble(sd);
        st.setAttributeValue(i, d);
        k++;
    }
    states.add(st);
    String debug = st.toString();
    System.out.println(debug);
}

public String toHeaderString() {
    String s = "";
    int i, j, m = State.getNumAttributes(), n = states.size();
    for (i = 0; i < n; i++)
        for (j = 0; j < m; j++)
            s += State.getAttributeName(j).charAt(0) + "." + (i + 1) + "," + "n";
    s = s.substring(0, s.length() - 2);
    s += "\n";
    return s;
}

@Override
public String toString() {
    String s = "";
    int i, j, m = State.getNumAttributes(), n = states.size();
    for (i = 0; i < n; i++)
        for (j = 0; j < m; j++)
            s += states.get(i).getAttributeValue(j) + ",\t";
    s = s.substring(0, s.length() - 2);
    s += "\n";
    return s;
}

package seven;
import com.csvreader.CsvReader;
import java.util.logging.Level;
import java.util.logging.Logger;

public class Connections {
    WeightedEdge[][] con;
    Double[] RowSums;
    Double[] ColSums;
    boolean normed=false;

    int Rows,Cols;
    Connections(int n){
        this.Rows=this.Cols=n;
        this.con = new WeightedEdge[Rows][Cols];
        this.ColSums = new Double[n];
        this.RowSums = new Double[n];
        for(int i=0;i<this.Rows;i++)
            this.RowSums[i]=new Double(0);
        for(int j=0;j<this.Cols;j++)
            this.ColSums[j]=new Double(0);
        for(int i=0;i<this.Rows;i++)
            for(int j=0;j<this.Cols;j++)
                this.con[i][j]=new WeightedEdge(new Integer(i),new Integer(j),new Double(0));

        try {
            readCSV();
        } catch (Exception ex) {
            // TODO Auto-generated catch block
            Logger.getLogger(Connections.class.getName()).log(Level.SEVERE, null, ex);
        }
        doSums();
    }
    Connections(int n,String dataFileName) {
        this.Rows=this.Cols=n;
        this.con = new WeightedEdge[Rows][Cols];
        this.ColSums = new Double[n];
        this.RowSums = new Double[n];
        for(int i=0;i<this.Rows;i++)
            this.RowSums[i]=new Double(0);
        for(int j=0;j<this.Cols;j++)
            this.ColSums[j]=new Double(0);
        for(int i=0;i<this.Rows;i++)
            for(int j=0;j<this.Cols;j++)
                this.con[i][j]=new WeightedEdge(new Integer(i),new Integer(j),new Double(0));

        try {
            readCSV(dataFileName);
        } catch (Exception ex) {
            // TODO Auto-generated catch block
            Logger.getLogger(Connections.class.getName()).log(Level.SEVERE, null, ex);
        }
        doSums();
    }
}
public Connections(WeightedEdge[][] con, int n) {
    this.Rows=this.Cols=n;
    for(int i=0;i<this.Rows;i++)
        for(int j=0;j<this.Cols;j++)
            this.con[i][j]=new WeightedEdge(con[i][j]);
}

public void readCSV(String sDataFileName) throws Exception {
    CsvReader reader = null;
    if(sDataFileName.equals(""))
        reader = new CsvReader("connections.csv");
    else
        reader = new CsvReader(sDataFileName);
    double d;
    String sd="";
    for(int i=0;i<Rows;i++)
    {
        reader.readRecord();
        for(int j=0;j<Cols;j++)
        {
            sd=reader.get(j);
        }
    }
    doSums();
}

public WeightedEdge getCon(int i,int j) {
    return con[i][j];
}

public void setCon(int i,int j,double value) {
    this.con[i][j].setWeight(new Double(value));
}

public int getSize() {
    return Rows;
}

public WeightedEdge getCon(int i, int j) {
    return con[i][j];
}

public void setCon(int i, int j, double value) {
    this.con[i][j].setWeight(new Double(value));
}

public int getSize() {
    return Rows;
}

public Connections(WeightedEdge[][] con, int n) {
    this.Rows = this.Cols = n;
    for (int i = 0; i < this.Rows; i++)
        for (int j = 0; j < this.Cols; j++)
            this.con[i][j] = new WeightedEdge(con[i][j]);
}

public void readCSV(String sDataFileName) throws Exception {
    CsvReader reader = null;
    if (sDataFileName.equals(""))
        reader = new CsvReader("connections.csv");
    else
        reader = new CsvReader(sDataFileName);
    double d;
    String sd = "";
    for (int i = 0; i < Rows; i++)
    {
        reader.readRecord();
        for (int j = 0; j < Cols; j++)
        {
            sd = reader.get(j);
        }
    }
    doSums();
}
d = Double.parseDouble(sd);
con[i][j] = new WeightedEdge(new Integer(i), new Integer(j), new Double(d));
}
}
reader.close();

private void doSums() {
    for (int i = 0; i < this.Rows; i++)
        for (int j = 0; j < this.Cols; j++) {
            Double d = this.con[i][j].getWeight();
            this.RowSums[i] += d;
            this.ColSums[j] += d;
        }
}

public Double getColSum(int c) {
    return ColSums[c];
}

public void setColSum(int c, Double d) {
    this.ColSums[c] = d;
}

public Double getRowSum(int r) {
    return RowSums[r];
}

public void setRowSum(int r, Double d) {
    this.RowSums[r] = d;
}

public boolean isNormed() {
    return normed;
}

public void setNormed(boolean normed) {
    this.normed = normed;
}
APPENDIX 2: MODEL EQUATION

Let $n$ be the number of states.
Let $m$ be the number of attributes.
Let $U$ be the $n \times m$ matrix of State-Attributes
Let $C$ be the connections between states. ($n \times n$)
Let $R$ be the diffusion between attributes ($m \times m$)
Let $\Delta U$ be the difference between the system at time $t +$ time $t$.
$U'$ is the subsequent system.

$$U' = U + C \circ \Delta U \circ R$$

or

$$u'_{ik} = u_{ik} + \sum_{j=1}^{n} \sum_{t=1}^{m} c_{ij} \delta_{jl} r_{lk}$$
APPENDIX 3: RESULTS FROM STATE WM

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Results of Test 1

Results of Test 2

Results of Test 3

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<tr>
<td>4</td>
<td>(4,3,0,0) (2,5,0,0) (0,7,0,0) (0,7,0,0) (0,7,0,0) (0,7,0,0)</td>
<td></td>
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<tr>
<td>5</td>
<td>(4,3,0,0) (4,3,0,0) (3,4,0,0) (2,5,0,0) (0,7,0,0) (0,7,0,0)</td>
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<tr>
<td>6</td>
<td>(4,3,0,0) (4,3,0,0) (4,3,0,0) (5,2,0,0) (5,2,0,0) (5,2,0,0)* (5,2,0,0)*</td>
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<tr>
<td>7</td>
<td>(4,3,0,0) (4,3,0,0) (5,2,0,0) (5,2,0,0) (5,2,0,0)* (5,2,0,0)*</td>
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</tbody>
</table>
Results of Test 8
**APPENDIX 4: RESULTS FROM STATE RA**

<table>
<thead>
<tr>
<th>Iteration</th>
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</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
<td>200</td>
<td>300</td>
<td>400</td>
<td>500</td>
</tr>
</tbody>
</table>

1: (4,3,0,0) (4,3,0,0) (4,3,0,0) (5,2,0,0) (5,2,0,0) (5,2,0,0) *

![Results of Test 9](image1)

2: (4,3,0,0) (4,3,0,0) (3,4,0,0) (2,5,0,0) (0,7,0,0) (0,7,0,0)

![Results of Test 10](image2)

3: (4,3,0,0) (3,4,0,0) (2,5,0,0) (0,7,0,0) (0,7,0,0) (0,7,0,0)

![Results of Test 11](image3)
4: \((4,3,0,0)\) \((0,7,0,0)\) \((0,7,0,0)\) \((0,7,0,0)\) \((0,7,0,0)\) \((0,7,0,0)\)*

Results of Test 12

---

5: \((4,3,0,0)\) \((4,3,0,0)\) \((3,4,0,0)\) \((2,5,0,0)\) \((0,7,0,0)\) \((0,7,0,0)\)

Results of Test 13

---

6: \((4,3,0,0)\) \((4,3,0,0)\) \((4,3,0,0)\) \((5,2,0,0)\) \((5,2,0,0)\) \((5,2,0,0)\)*

Results of Test 14

---

7: \((4,3,0,0)\) \((4,3,0,0)\) \((5,2,0,0)\) \((5,2,0,0)\) \((5,2,0,0)\) \((5,2,0,0)\)*

Results of Test 15

---
8: (4,3,0,0) (5,2,0,0) (5,2,0,0) (5,2,0,0) (5,2,0,0) (5,2,0,0) (5,2,0,0)
REFERENCES


